INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MATHEMATICS
HIGHER LEVEL
PAPER 1

Thursday 2 November 2000 (afternoon)

2 hours

Name

Number

INSTRUCTIONS TO CANDIDATES

- Write your name and candidate number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.
- Write the make and model of your calculator in the box below *e.g.* Casio *fx-7400G*, Sharp EL-9400, Texas Instruments TI-80.

Calculator

Make	Model

EXAMINER	TEAM LEADER	IBCA
TOTAL /60	TOTAL /60	TOTAL /60

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary. Where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.

1. Find the values of the real number k for which the determinant of the matrix $\begin{pmatrix} k-4 & 3 \\ -2 & k+1 \end{pmatrix}$ is equal to zero.

Working:

Answers:

2. Given functions $f: x \mapsto x + 1$ and $g: x \mapsto x^3$, find the function $(f \circ g)^{-1}$.

Working:

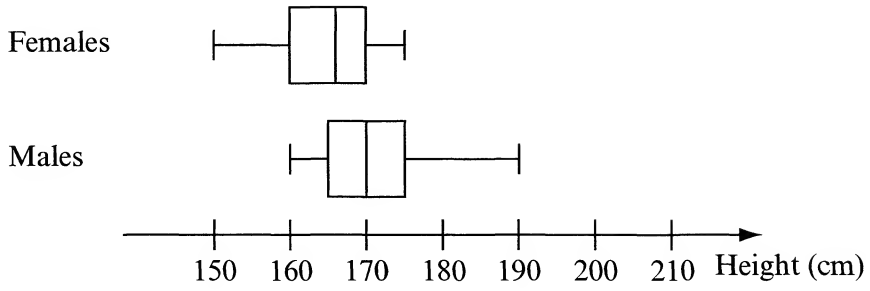
Answer:

3. For the function f: $x \mapsto x^2 \ln x$, $x > 0$, find the function f', the derivative of f with respect to x.

Working:

Answer:

4. The box-and-whisker plots shown represent the heights of female students and the heights of male students at a certain school.



(a) What percentage of female students are shorter than any male students?

(b) What percentage of male students are shorter than some female students?

(c) From the diagram, **estimate** the mean height of the male students.

Working:

Answers:

(a) _____

(b) _____

(c) _____

5. Calculate the area bounded by the graph of $y = x \sin (x^2)$ and the x-axis, between $x = 0$ and the smallest positive x-intercept.

Working:

Answer:

6. Given that events A and B are independent with $P(A \cap B) = 0.3$ and $P(A \cap B') = 0.3$, find $P(A \cup B)$.

Working:

Answer:

7. Find the sum of the positive terms of the arithmetic sequence 85 , 78 , 71 , ...

Working:

Answer:

8. For the function $f : x \mapsto \frac{1}{2} \sin 2x + \cos x$, find the possible values of $\sin x$ for which $f'(x) = 0$.

Working:

Answers:

9. (a) Describe the transformation of the plane whose matrix is

$$\boldsymbol{M} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}.$$

(b) Find the smallest positive integer n for which $\boldsymbol{M}^n = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Working:

Answers:

(a) _____

_____

(b) _____

10. Find the real number k for which $1 + ki$, $(i = \sqrt{-1})$, is a zero of the polynomial $z^2 + kz + 5$.

Working:

Answer:

11. Let α be the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, where $\boldsymbol{a} = (\cos\theta)\boldsymbol{i} + (\sin\theta)\boldsymbol{j}$, $\boldsymbol{b} = (\sin\theta)\boldsymbol{i} + (\cos\theta)\boldsymbol{j}$ and $0 < \theta < \frac{\pi}{4}$.

Express α in terms of θ.

Working:

Answer:

12. The coefficient of x in the expansion of $\left(x + \frac{1}{ax^2}\right)^7$ is $\frac{7}{3}$. Find the possible values of a.

Working:

Answers:

13. For what values of m is the line $y = mx + 5$ a tangent to the parabola $y = 4 - x^2$?

Working:

Answers:

14. The tangent to the curve $y^2 = x^3$ at the point P(1 , 1) meets the x-axis at Q and the y-axis at R . Find the ratio PQ : QR .

Working:

Answer:

15. The sum of an infinite geometric sequence is $13\frac{1}{2}$, and the sum of the first three terms is 13. Find the first term.

Working:

Answer:

16. In a triangle ABC, $\widehat{ABC} = 30°$, AB = 6 cm and $AC = 3\sqrt{2}$ cm. Find the possible lengths of [BC].

Working:

Answers:

17. Solve the differential equation $xy\dfrac{dy}{dx} = 1 + y^2$, given that $y = 0$ when $x = 2$.

Working:

Answer:

18. If z is a complex number and $|z + 16| = 4|z + 1|$, find the value of $|z|$.

Working:

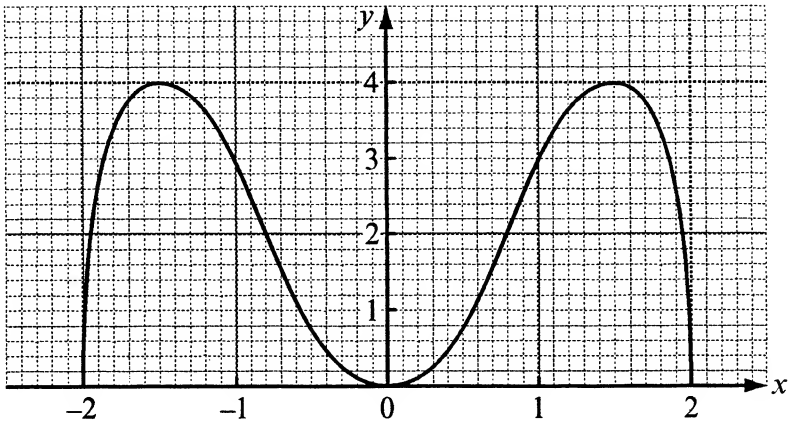
Answer:

19. In how many ways can six different coins be divided between two students so that each student receives at least one coin?

Working:

Answer:

20. The following graph is that part of the graph of $y = f(x)$ for which $f(x) \geq 0$.



Sketch, on the axes provided below, the graph of $y^2 = f(x)$ for $-2 \leq x \leq 2$.